Midterm: Avg 62.8%

- Lecture survey – new Q's about midterm study materials
- Mark the date: Final Dec 15

Last time: Exponential growth, e^x (inverses) $\ln(x)$

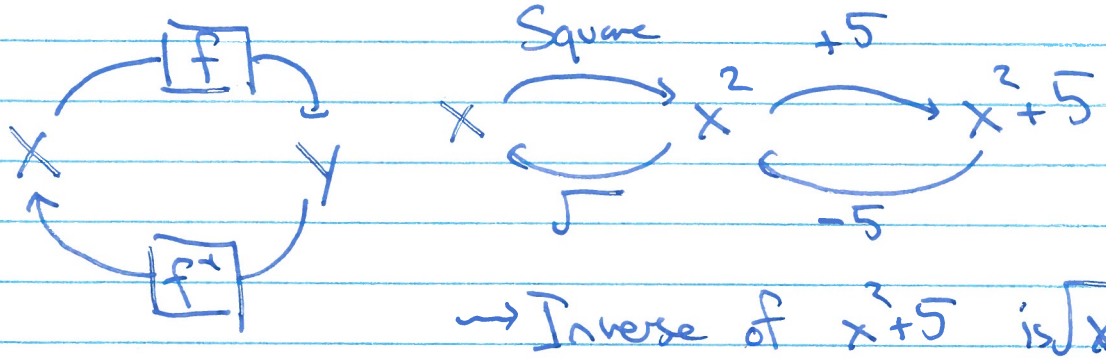
x. Bacterial colony, grows exponentially
- At t=0 100 cells
- At t=3 500 cells

Q: How many cells at t=7? Q: What time t were there 200 cells

Today: - Review of inverse functions
- Derive properties of logarithms.
- Use these to answer the question.

Inverses

	$f(x)$	$f^{-1}(x)$	
Add	$x+3$	$x-3$	Subtract
Multiply	$5x$	$x/5$	Divide
Power	x^a	$x^{1/a}$	Root $\rightsquigarrow (x^a)^{\frac{1}{a}} = x^{(a\cdot\frac{1}{a})}$
	e^x	$\ln(x)$	$\rightsquigarrow \ln(e^x) = x \quad = x^1$
	2^x	$\log_2(x)$	



→ Inverse of x^2+5 is $\sqrt{x-5}$

Rules of exponentiation (True for ANY base. Not just e.)

$$e^a \cdot e^b = e^{a+b}$$

Ex. $e^2 \cdot e^3 = (e\cdot e)(e\cdot e\cdot e)$

$= e\cdot e\cdot e\cdot e\cdot e$

$= e^5$

$$(e^a)^b = e^{ab}$$

Ex. $(e^2)^3 = (e\cdot e)(e\cdot e)(e\cdot e)$

$= e\cdot e\cdot e\cdot e\cdot e\cdot e$

$= e^6$

Rules for logarithm.

Take $\ln(\cdot)$

$$\ln(e^a \cdot e^b) = \ln(e^{a+b})$$

$$\ln(e^a \cdot e^b) = a+b$$

Let $A = e^a$, $B = e^b$

then $\ln(A\cdot B) = \ln(A) + \ln(B)$

$$\ln(A^b) = b\ln(A)$$

Logarithms in other bases: "$\log_c(x)$" answers the question "What number must I raise c to to get x"

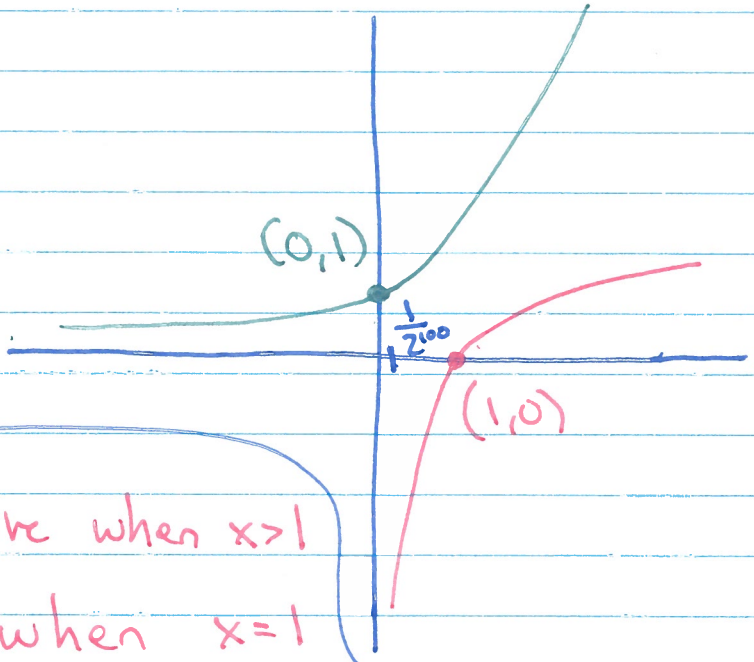
Ex: $\log_2 8 = 3$

$$\log_c(x) = \frac{\ln(x)}{\ln(c)}$$

Interesting note: $y = f(x)$ & $y = f^{-1}(x)$ are reflections over $y = x$

Ex $y = e^x$

$y = \ln(x)$



$\ln(x)$ is { positive when $x > 1$
0 when $x = 1$
negative when $0 < x < 1$

	Domain	Range
e^x	$(-\infty, \infty)$	$(0, \infty)$
$\ln(x)$	$(0, \infty)$	$(-\infty, \infty)$

Ex. Bacterial colony grows at a rate proportional to size

$B(t)$ = # of bacteria @ time t.

$B'(t) = kB(t)$

Therefore $B(t) = Ce^{kt}$ for some constants C, k.

I. At $t=0$, $B(0) = 100$

II. At $t=3$, $B(3) = 500$.

Q: At $t=7$, how many cells.

Q2: What t gives 200 cells?

Q1: $B(7) = \boxed{100 \cdot 5^{7/3}} = 100 \cdot e^{\ln 5 \cdot \frac{7}{3}}$

"After 3 hours, size $\times 5$."

$B(t) = 100 \cdot 5^{t/3}$

start with 100 cells.

Q2: $100 \cdot 5^{t/3} = 200 \rightsquigarrow 5^{t/3} = 2$

$\downarrow \log_5$

$t/3 = \log_5 2 = \frac{\ln 2}{\ln 5}$

$\downarrow$

$\boxed{t = 3\ln 2 / \ln 5}$